Lecture 2 (Nov 12)

A. Trig. Review

Euler's formula: $e^{jx} = \cos x + j\sin x$

$$e^{j(-x)} = \cos(-x) + j\sin(-x) = \cos(x) - j\sin(x)$$

$+$

$$e^{jx} + e^{-jx} = 2\cos x$$

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

Example: $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

To see this,

$$\cos^2 x = \left(\frac{1}{2}\left(e^{jx} + e^{-jx}\right)\right)^2$$

$(a+b)^2$

$$= \frac{1}{4}\left(e^{2jx} + 2\underbrace{e^{jx}e^{-jx}} + e^{-2jx}\right)$$

$$= \frac{1}{4}\left(\underbrace{e^{2jx} + e^{-2jx}} + 2\right)$$

$$= \frac{1}{4}\left(2\cos 2x + 2\right)$$

$$= \frac{1}{2}\left(\cos 2x + 1\right)$$

Similar technique gives

$$\cos x \times \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$

Please show this in HW 1.

(Q1 of HW1) | Important!

B: Fourier Transform

Definition

D1 $$X_1(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \quad \text{(use } f\text{)}$$

D2 $$X_2(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad \text{(use } \omega\text{)}$$

Q: Given same $x(t)$,

what is the relationship btw X_1 and X_2?

A: $$\left.\begin{aligned} X_1(f) &= X_2(2\pi f) \\ X_2(\omega) &= X_1\left(\frac{\omega}{2\pi}\right) \end{aligned}\right\} \quad \begin{aligned} \omega &= 2\pi f \\ f &= \frac{\omega}{2\pi} \end{aligned}$$

Example: Suppose we know that

$$e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi \delta(\omega - \omega_0)$$

same

$$\omega_0 = 2\pi f_0, \quad \omega = 2\pi f$$

$$e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} 2\pi \delta(2\pi f - 2\pi f_0)$$

$$= \delta(f - f_0)$$

Trick: $\delta(a \times t) = \frac{1}{|a|} \delta(t)$

Inverse Transform

$$x(t) = \int_{-\infty}^{\infty} X_1(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} \underbrace{X_1\left(\frac{\omega}{2\pi}\right)} e^{j\omega t} d\frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_2(\omega) e^{j\omega t} d\omega$$

P1 If $x(t)$ is real-valued, then

conjugate

$$x^*(t) = x(t)$$

$$X(-f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi(-f)t}\, dt$$

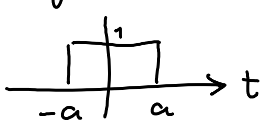
$$= \int_{-\infty}^{\infty} x^*(t) \left(e^{-j2\pi f t}\right)^* dt$$

$$= \left(\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt\right)^* = \left(X(f)\right)^*$$

conclusion : $X(-f) = \left(X(f)\right)^*$

Interpretation : Only half of the spectrum contain <u>all</u> of the info. !!

(P2) Rectangular function $\xrightarrow{\mathcal{F}}$ sinc function



$$2a \times \frac{\sin(2\pi f a)}{2\pi f a} = 2a\,\mathrm{sinc}(2\pi f a)$$

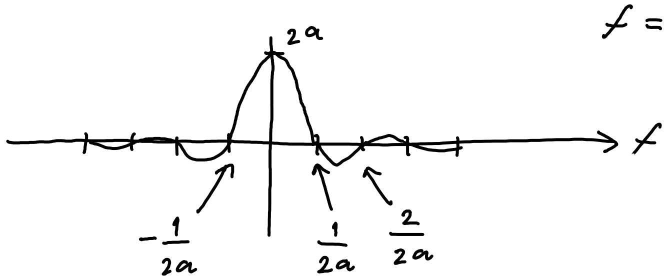
$$\mathrm{sinc}\, x = \frac{\sin x}{x}$$

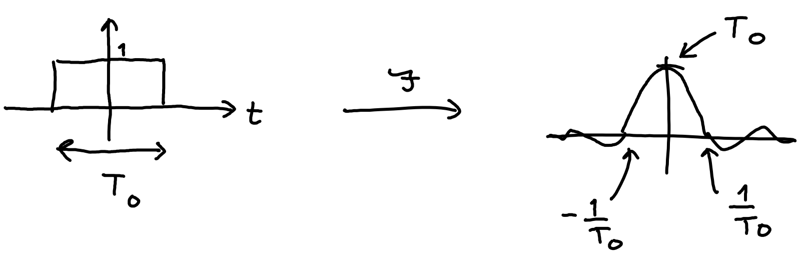
$$\frac{\sin 2\pi f a}{\pi f}$$

$$\sin x = 0$$

$$x = n\pi = 2\pi f a$$

$$f = \frac{n\cancel{\pi}}{2\cancel{\pi} a}$$

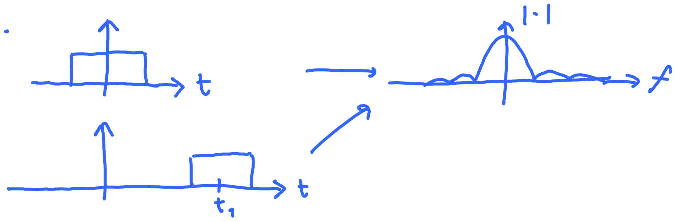




(P3) Time - Shift

$$g(t-t_1) \xrightarrow{\mathcal{F}} \underbrace{e^{-j2\pi f t_1}}_{|\cdot| = 1} G(f)$$

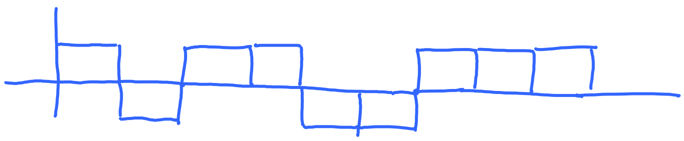
Ex.



Lecture 3 (Nov 16)

- Tuesday class moved to 1PM - 2:20 PM
- HW 1 (due next Tuesday)

Ex.



$$\sum_{k=1}^{n} m_k \underbrace{g(t-\tau_k)} \xrightarrow{\mathcal{F}} \sum_{k=1}^{n} m_k e^{-j2\pi f \tau_k} G(f)$$

$$|\cdot| = \left| \left(\sum_k m_k e^{-j2\pi f \tau_k} \right) G(f) \right|$$

$$= \left| \sum_{k=1}^{n} m_k e^{-j2\pi f \tau_k} \right| |G(f)|$$

last term

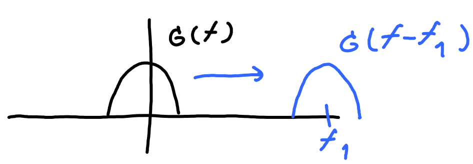
$$e^{-j2\pi f \tau_n}$$

$$= \cos(2\pi f \tau_n) - j \sin(2\pi f \tau_n)$$

$$= \cos(2\pi \tau_n f) - j \sin(2\pi \tau_n f)$$

Frequency Shift property

$$e^{j2\pi f_1 t} g(t) \xrightarrow{\mathcal{F}} G(f - f_1)$$



Convolution

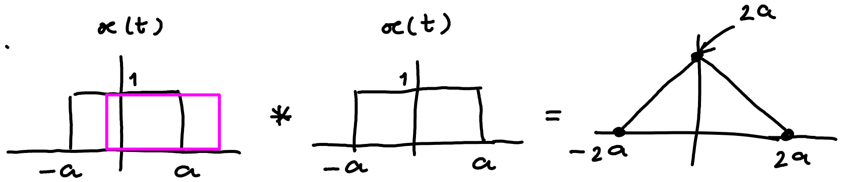
Two signals/wave form $x(t)$, $y(t)$

The convolution of x and y are given by

$$z(t) = x(t) * y(t) = \{x * y\}(t)$$

$$= \int_{-\infty}^{\infty} x(\mu)\, y(t-\mu)\, d\mu = \int_{-\infty}^{\infty} x(t-\mu)\, y(\mu)\, d\mu$$

Ex.



$$g_1 * g_2 \xrightarrow{\mathcal{F}} G_1 \times G_2$$

HW1 Q2